HEAT AND MASS TRANSFER IN A BINARY TURBULENT BOUNDARY LAYER WITH NATURAL CONVECTION AT VERTICAL SURFACE

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An approximate solution is given for binary turbulent natural convection, with allowance for diffusive heat conduction. Analytic expressions are obtained for the boundary-layer outlet characteristics (coefficients of heat exchange and mass transfer). The solution is compared with experiments on heat and mass transfer during condensation of vapors from mixtures with various gases.

We write the integral equation of momentum for a binary turbulent layer developing in the presence of gravitational convection at a vertical surface as

$$\frac{d}{dx}\int_0^h u^2 dy = -\frac{\tau_w}{\rho_\infty} + \int_0^h \frac{\rho-\rho_\infty}{\rho_\infty} g dy. \tag{1}$$

The change in density over the boundary-layer thickness can be written, for a binary mixture, as the sum of two parts: the variations caused by temperature, and the variations associated with the gradient of mass concentration for the "active" component "1":

$$\frac{\rho-\rho_\infty}{\rho_\infty} = -\beta_t (t-t_\infty) - \beta_m (m_1 - m_{1\infty}), \tag{2}$$

where the coefficient of volume concentration expansion for an ideal binary mixture is found from the relationship

$$\beta_m = \frac{M_2/M_1 - 1}{1+(M_2/M_1-1)\, m_1}. \tag{3}$$

The output characteristics of a turbulent boundary layer are conservative with respect to the porous supply of matter; thus we assume that the tangential stress at the wall is qualitatively analogous to the stress for turbulent natural convection at impenetrable surfaces [1],

$$\tau_w = 0.253\rho_w u_1^2 \left(\frac{\nu}{u_1\delta}\right)^{1/2}. \tag{4}$$

Qualitatively, we should expect significant differences, since u_1 and δ are found by simultaneous solution of the equations of motion, energy, and diffusion for the binary boundary layer, where the characteristics of penetrability must affect the values of u_1 and δ.

Substituting (2) and (4) into (1) and going from the thickness h to the thicknesses of the hydrodynamic (δ) and diffusion (δ_m) boundary layers, we have

$$\frac{d}{dx}\int_0^\delta u^2 dy = -\beta_t g\int_0^\delta (t-t_\infty)\, dy - \beta_m g \int_0^{\delta_m} (m_1 - m_{1\infty}) dy - 0.253\bar{\rho} u_1^2 \left(\frac{\nu}{u_1\delta}\right)^{1/2}. \tag{5}$$

We write the equation for the energy of the binary turbulent boundary layer as

$$\frac{d}{dx}\int_0^\delta u(t-t_\infty)\, dy = \frac{q_{Tw}}{g\rho_\infty c_p} + \frac{a_T R M^2 T_w}{427 M_1 M_2 c_p \rho_\infty} j_{1w} + \bar{\rho} v_w (t_w - t_\infty), \tag{6}$$

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where the sum of the first two terms on the right side equals the heat flux resulting from conduction (q_{Tw}) and the diffusion conduction

$$q_w = q_{\tau w} + \frac{a_\tau R M^2 T_w}{427 M_1 M_2} j_{1w}. \quad (7)$$

For the heat flux at the wall that is transmitted by conduction, we use an expression resembling that for an impenetrable surface:

$$q_{\tau w} = 0.253 g \rho_w c_p u_1 \left(\frac{\nu}{u_1 \delta}\right)^{1/2} (t_w - t_\infty) \,\mathrm{Pr}^{-2/3}, \quad (8)$$

obtained on the basis of the Reynolds analogy in [1]; here the penetrability characteristics affect the values of u_1 and δ, found by simultaneous solution of the equations of motion, energy, and diffusion. The factor $\mathrm{Pr}^{-2/3}$ allows for the deviation from the exact analogy when Pr differs from unity. Then (6) can be written in the form

$$\frac{d}{dx}\int_0^\delta u(t - t_\infty)\, dy = 0.253 \bar{\rho} u_1 \left(\frac{\nu}{u_1 \delta}\right)^{1/2} (t_w - t_\infty)\,\mathrm{Pr}^{-2/3} + \frac{a_\tau R M^2 T_w}{427 M_1 M_2 c_p \rho_\infty} j_{1w} + \bar{\rho} v_w (t_w - t_\infty). \quad (9)$$

The integral equation of diffusion has the form

$$\frac{d}{dx}\int_0^{\delta_m} u(m_1 - m_{1\infty})\, dy = \frac{j_{1w}}{\rho_\infty} + \bar{\rho} v_w (m_{1w} - m_{1\infty}). \quad (10)$$

Neglecting the mass flow resulting from thermal diffusion and using the Colborn analogy between momentum and diffusion, we determine the concentration flow of component "1" at the wall,

$$j_{1w} = \frac{\tau_w}{u_1} \mathrm{Sc}^{-2/3} (m_{1w} - m_{1\infty}), \quad (11)$$

where the factor $\mathrm{Sc}^{-2/3}$ allows for the deviation from the exact analogy when the Schmidt number differs from unity.

Allowing for (4), we can represent the concentration mass flow of component "1" as

$$j_{1w} = 0.253 \rho_w u_1 \left(\frac{\nu}{u_1 \delta_m}\right)^{1/2} \mathrm{Sc}^{-2/3} (m_{1w} - m_{1\infty}). \quad (12)$$

The transverse velocity v_w at the wall is associated with the concentration flow of component "1" by the relationship

$$v_w = \frac{j_{1w}}{\rho_w (1 - m_{1w})}. \quad (13)$$

To solve the system (5), (9), and (10), we must specify the distributions of velocity, temperature, and mass concentration of component "1" in the boundary layer; we use the "one-seventh" law:

$$u = u_1 \left(\frac{y}{\delta}\right)^{1/7} \left(1 - \frac{y}{\delta}\right)^4; \quad t - t_\infty = (t_w - t_\infty)\left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right];$$
$$m_1 - m_{1\infty} = (m_{1w} - m_{1\infty})\left[1 - \left(\frac{y}{\delta_m}\right)^{1/7}\right]. \quad (14)$$

Solving (5), (9), and (10), with allowance for (13) and (14), we have

$$\frac{\mathrm{Nu}_x}{\mathrm{Nu}_{x_0}} = \bar{\rho}^{-1/3}\left[1 + \frac{\beta_m}{\beta_t}\frac{m_{1\infty} - m_{1w}}{t_\infty - t_w}\frac{1}{\xi}\right]^{1/3}$$
$$\times\left\{1 + \frac{2{,}14}{2{,}14 + \mathrm{Pr}^{2/3}}\left(\frac{\mathrm{Pr}}{\mathrm{Sc}}\right)^{2/3}\frac{m_{1w} - m_{1\infty}}{1 - m_{1w}}\left[1 + \mathrm{Du}\,(1 - m_{1w})\right]\xi^{1/2}\right\}^{-1/3}\left[1 + \left(\frac{\mathrm{Pr}}{\mathrm{Sc}}\right)^{2/3}\mathrm{Du}\,(m_{1w} - m_{1\infty})\,\xi^{1/2}\right], \quad (15)$$

$$\frac{\mathrm{Sh}_x}{\mathrm{Nu}_x} = \frac{\left(\frac{\mathrm{Sc}}{\mathrm{Pr}}\right)^{1/3}\xi^{1/2}}{(1 - m_{1w})\left[1 + \left(\frac{\mathrm{Pr}}{\mathrm{Sc}}\right)^{2/3}\mathrm{Du}\,(m_{1w} - m_{1\infty})\,\xi^{1/2}\right]}, \quad (16)$$

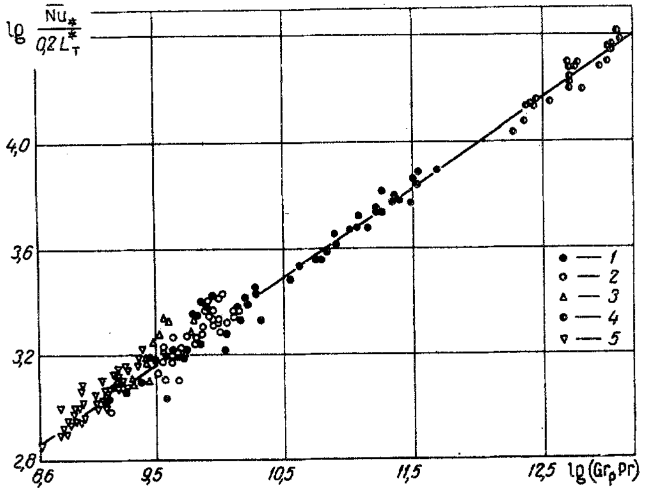

Fig. 1. "Total" heat exchange (with allowance for heat of phase transformations) in condensation of various vapors from mixtures with noncondensing gases in the presence of turbulent natural convection at vertical surfaces (the curve is based on (22) or (24)): 1-4) experiments of Mazyukevich [2, 3] [1) NH_3–air; l = 276 and 500 mm; $\varepsilon_{va}^{\infty}$ = 2-35%; 2) NH_3–H_2, l = 276 mm, $\varepsilon_{vH_2}^{\infty}$ = 5-33%; 3) NH_3–CH_4, l = 276 mm, $\varepsilon_{vCH_4}^{\infty}$ = 19.5-33%; 4) Freon-12–air, l = 500 mm, $\varepsilon_{va}^{\infty}$ = 14.7-32.6%]; 5) experiments of [4], H_2O–air, $\varepsilon_{va}^{\infty}$ = 89-98%, $L_T^* = \overline{\rho}^{-1/3}\left(\frac{\mathrm{Pr}^{2/3}}{2,14+\mathrm{Pr}^{2/3}}\right)^{1/3}\left\{1+\frac{2,14}{2,14+\mathrm{Pr}^{2/3}}\left(\frac{\mathrm{Pr}}{\mathrm{Sc}}\right)^{2/3}\frac{m_{1w}-m_{1\infty}}{1-m_{1w}}[1+\mathrm{Du}(1-m_{1w})]\,\xi^{1/2}\right\}^{-1/3}\left[1+\left(\frac{\mathrm{Pr}}{\mathrm{Sc}}\right)^{2/3}(m_{1w}-m_{1\infty})\left(\mathrm{Du}+\frac{\mathrm{K}}{1-m_{1w}}\right)\xi^{1/2}\right]$.

where

$$\mathrm{Nu}_{x_0} = 0.2\,(\mathrm{Gr}_x\,\mathrm{Pr})^{1/3}\left(\frac{\mathrm{Pr}^{2/3}}{2,14+\mathrm{Pr}^{2/3}}\right)^{1/3} \tag{17}$$

is the local Nusselt number at an impenetrable surface under conditions of turbulent natural convection [1].

The mass-transfer coefficient was computed from the total mass flow of component "1," which is associated with the concentration flow by the expression

$$W_{1w} = \frac{j_{1w}}{1-m_{1w}} = \alpha_m\rho_w\,(m_{1w}-m_{1\infty}). \tag{18}$$

In (15) and (16), the ratio of the boundary layers $\xi = \delta/\delta_m$ was found, depending on whether ξ is greater or less than unity, from the following relationships:

a) $0.1 \le \xi \le 1.0$

$$\xi = 0.309\left\{\frac{m_{1w}-m_{1\infty}}{1-m_{1\infty}}[\mathrm{Du}(1-m_{1w})+1]+\sqrt{\left\{\frac{m_{1w}-m_{1\infty}}{1-m_{1\infty}}[\mathrm{Du}(1-m_{1w})+1]\right\}^2+3.6\left(\frac{1-m_{1w}}{1-m_{1\infty}}\,\mathrm{Le}^{2/3}-0,1\right)}\right\}^2; \tag{19}$$

b) $1.0 \le \xi \le 2.0$

$$\xi = \left\{\frac{1}{2}\,\frac{m_{1w}-m_{1\infty}}{1-m_{1\infty}}[\mathrm{Du}(1-m_{1w})+1]+\sqrt{\frac{1}{4}\left\{\frac{m_{1w}-m_{1\infty}}{1-m_{1\infty}}[\mathrm{Du}(1-m_{1w})+1]\right\}^2+\frac{1-m_{1w}}{1-m_{1\infty}}\,\mathrm{Le}^{2/3}}\right\}^2. \tag{20}$$

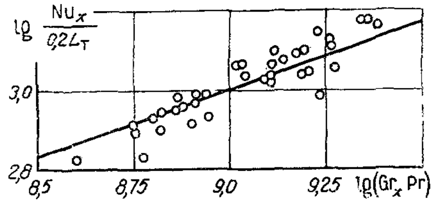

Fig. 2. Convective local heat exchange for condensation of water vapor from moist air with turbulent natural convection at vertical surface (curve corresponds to formula (15), and the circles to experiment [4]; $\varepsilon_{va}^{\infty}$ = 89-98%). $L_T = \bar{\rho}^{-1/3} \left(\frac{Pr^{2/3}}{2,14+Pr^{2/3}}\right)^{1/3} \left[1+\frac{\beta_m}{\beta_t}\,\frac{m_{1\infty}-m_{1w}}{t_\infty-t_w}\,\frac{1}{\xi}\right]^{1/3} \left\{1+\frac{2,14}{2,14+Pr^{2/3}} \left(\frac{Pr}{Sc}\right)^{2/3} \frac{m_{1w}-m_{1\infty}}{1-m_{1w}} [1+Du(1-m_{1w})]\,\xi^{1/2}\right\}^{-1/3} \left[1+\left(\frac{Pr}{Sc}\right)^{2/3} Du(m_{1w}-m_{1\infty})\,\xi^{1/2}\right]$.

When there are phase transformations or chemical reactions at the wall, the "total" heat flux through the wall to the cooling medium is found from the formula

$$q_w^* = q_w + rW_{1w}. \tag{21}$$

We next use (21) to find the arbitrary heat-exchange coefficient with allowance for the heat α^* of the phase transformations or chemical reactions:

$$\frac{Nu_x^*}{Nu_{x0}} = \bar{\rho}^{-1/3}\left[1+\frac{\beta_m}{\beta_t}\,\frac{m_{1\infty}-m_{1w}}{t_\infty-t_w}\,\frac{1}{\xi}\right]^{1/3} \left\{1+\frac{2,14}{2,14+Pr^{2/3}}\left(\frac{Pr}{Sc}\right)^{2/3}\frac{m_{1w}-m_{1\infty}}{1-m_{1w}} \times [1+Du(1-m_{1w})]\,\xi^{1/2}\right\}^{-1/3} \left[1+\left(\frac{Pr}{Sc}\right)^{2/3}(m_{1w}-m_{1\infty}) \times\left(Du+\frac{K}{1-m_{1w}}\right)\xi^{1/2}\right]. \tag{22}$$

Figure 1 compares the solution (22) with the experimental data of Mazyukevich [2, 3] on mean heat exchange in condensation of ammonia vapors from a mixture with hydrogen, methane, and air, and also Freon-12 from a mixture with air in vertical tubes of height 276 and 500 mm. The figure also shows our experimental data for condensation of water vapor from moist air [4]. For the generalization of Fig. 1, the ranges over which the fundamental parameters vary are as follows: volume content of inert gas in mixture, $\varepsilon_{vi}^{\infty}$ = 2-98%; $Gr_\rho = 5.5\cdot10^8$-$7.2\cdot10^{12}$; Pr = 0.7-2.15; Le = 0.17-1.0; Du = (–58)-2; the Kutateladze number K = 11-480; the mixture pressure p_m = 1.0-13.6 atm abs.

In the processing of the experimental data, the characteristic temperature and concentration were taken to be the temperature and concentration at a large distance from the condensation surface (t_∞, $m_{1\infty}$). The partial vapor pressure at the wall was found from the wall temperature. The relationship between the partial vapor pressure in the mixture and its mass content was established from the formula

$$m_1 = \frac{R_2 p_v}{R_1 p_m - p_v(R_1-R_2)},$$

where p_m is the mixture pressure, and p_v is the partial vapor pressure.

Where the content of the inert gas in the vapor–gas mixture is low, formula (3) for the coefficient of volume concentration expansion is incorrect. Thus in (22) we took $[1 + (\beta_m/\beta_t)(m_{1\infty}-m_{1w})/(t_\infty-t_w)\,(1/\xi)] = 1$, while the Grashof number of the mixture was found from the difference of the densities,

$$Gr_\rho = \frac{gl^3}{v^2}\left|\frac{\rho_w-\rho_\infty}{\rho_\infty}\right|.$$

As a result, (15) and (22) take the form

$$Nu_x = 0,2\,(Gr_{x\rho}\,Pr)^{1/3}\left(\frac{Pr^{2/3}}{2,14+Pr^{2/3}}\right)^{1/3}\bar{\rho}^{-1/3}\left\{1+\frac{2,14}{2,14+Pr^{2/3}}\left(\frac{Pr}{Sc}\right)^{2/3} \times\frac{m_{1w}-m_{1\infty}}{1-m_{1\infty}}[1+Du(1-m_{1w})]\,\xi^{1/2}\right\}^{-1/3}\left[1+\left(\frac{Pr}{Sc}\right)^{2/3}Du(m_{1w}-m_{1\infty})\,\xi^{1/2}\right], \tag{23}$$

$$Nu_x^* = 0,2\,(Gr_{x\rho}\,Pr)^{1/3}\left(\frac{Pr^{2/3}}{2,14+Pr^{2/3}}\right)^{1/3}\bar{\rho}^{-1/3} \times\left\{1+\frac{2,14}{2,14+Pr^{2/3}}\left(\frac{Pr}{Sc}\right)^{2/3}\frac{m_{1w}-m_{1\infty}}{1-m_{1w}}[1+Du(1-m_{1w})]\,\xi^{1/2}\right\}^{-1/3} \times\left[1+\left(\frac{Pr}{Sc}\right)^{2/3}(m_{1w}-m_{1\infty})\left(Du+\frac{K}{1-m_{1w}}\right)\xi^{1/2}\right]. \tag{24}$$

The thermal conductivity, viscosity, diffusion coefficient, and thermal-diffusion constant were found on the basis of the modified Buckingham potential [5]. Thus, for example, the thermal-diffusion constant for mixtures varied, depending on the temperature and the mass content of vapor in the mixtures as follows: NH_3-H_2, $\alpha_T = 0.14\text{-}0.16$; NH_3–air, $\alpha_T = (-0.051)\text{-}(-0.045)$; NH_3-CH_4, $\alpha_T \approx -0.002$; Freon-12–air, $\alpha_T = 0.016\text{-}0.019$; H_2O–air, $\alpha_T = (-0.08)\text{-}(-0.09)$.

Figure 2 shows the local convective heat exchange for condensation of water vapor from moist air at a vertical plate [4]. The theoretical curve (Fig. 2) corresponds to formula (15). The volume content of water vapor in air varied for the experiments of [4] within the range $\varepsilon_{vv}^{\infty} = 2\text{-}11\%$.

For the experiments covered by Figs. 1 and 2, the maximum pumping rates were as follows: for the H_2O–air mixture, $v_w^{max} = 0.1$ m/sec; NH_3–air, $4.4 \cdot 10^{-3}$ m/sec; NH_3-H_2, $3.8 \cdot 10^{-3}$ m/sec; Freon-12–air, $3.5 \cdot 10^{-3}$ m/sec; NH_3-CH_4, 3.0 m/sec.

NOTATION

x, y	are the longitudinal and transverse coordinates;
u, v	are the longitudinal and transverse velocity components;
t	is the temperature;
T	is the absolute temperature;
m_i	is the mass proportion of the i-th component;
ε_{vi}	is the volume content of inert gas;
ε_{vv}	is the volume content of vapor in the mixture;
β_t, β_m	are the thermal and concentration coefficients of volume expansion;
ρ	is the density;
$\bar{\rho} = \rho_w/\rho_\infty$;	
c_p	is the isobaric heat capacity;
ν	is the coefficient of kinematic viscosity;
a	is the thermal diffusivity;
D	is the diffusion coefficient;
α_T	is the thermal-diffusion constant;
λ	is the coefficient of thermal conductivity;
M	is the molecular weight;
R	is the gas constant;
r	is the phase-transition heat;
τ_w	is the tangential stress at the wall;
q	is the convective heat flux;
q_w^*	is the heat flux with allowance for phase transformation or chemical reactions at the wall;
q_T	is the heat flux due to heat conduction;
j_i	is the diffusion flow of the i-th component;
W_i	is the total mass flow of the i-th component;
δ	is the width of the thermal boundary layer;
δ_m	is the thickness of the diffusion boundary layer;
$\xi = \delta/\delta_m$;	
α	is the coefficient of convective heat exchange;
α^*	is the arbitrary heat-exchange coefficient with allowance for the heat of phase transformations;
Nu	is the convective Nusselt number;
Nu*	is the Nusselt number constructed for α^*;
$Sc = \nu/D$	is the Schmidt number;
Pr	is the Prandtl number;
$Le = D/a$	is the Lewis number;
$Du = [(\alpha_T R M^2)/(427 M_1 M_2 c_p)] \cdot [T_w/(t_w - t_\infty)]$	is the Duffour number;
$Sh = \alpha_m x/D$	is the Sherwood number;
$K = r/c_p(t_w - t_\infty)$	is the Kutateladze number;
$Gr_x = g\beta_t(t_w - t_\infty)x^3/\nu^2$	is the Grashof number, taken for the temperature difference;
$Gr_{x\rho} = [(gx^3)/\nu^2] \cdot \lvert[(\rho_w - \rho_\infty)/\rho_\infty]\rvert$	is the Grashof number taken for the density difference.

Subscripts

w	is the wall;
∞	is the outside wall;
0	is the value at an impenetrable wall;
1	is the active component of binary mixture (vapor);
2	is the inert component of mixture;
x	is the local value;
overscore	is the average value;
a	is the air;
v	is the vapor.

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